Exercise 38

An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the coefficient of friction.

(a) Find the rate of change of F with respect to θ.

(b) When is this rate of change equal to 0?

(c) If $W = 50$ lb and $\mu = 0.6$, draw the graph of F as a function of θ and use it to locate the value of θ for which $dF/d\theta = 0$. Is the value consistent with your answer to part (b)?

Solution

The rate of change of F with respect to θ is

$$\begin{aligned}
\frac{dF}{d\theta} &= \frac{d}{d\theta}\left(\frac{\mu W}{\mu \sin\theta + \cos\theta}\right) \\
&= \frac{\left[\frac{d}{d\theta}(\mu W)\right](\mu \sin\theta + \cos\theta) - \left[\frac{d}{d\theta}(\mu \sin\theta + \cos\theta)\right](\mu W)}{(\mu \sin\theta + \cos\theta)^2} \\
&= \frac{(0)(\mu \sin\theta + \cos\theta) - (\mu \cos\theta - \sin\theta)(\mu W)}{(\mu \sin\theta + \cos\theta)^2} \\
&= -\mu W \frac{\mu \cos\theta - \sin\theta}{(\mu \sin\theta + \cos\theta)^2}
\end{aligned}$$

This rate of change is zero when the numerator is zero.

$$\frac{dF}{d\theta} = 0 \quad \Rightarrow \quad \mu \cos\theta - \sin\theta = 0 \quad \rightarrow \quad \tan\theta = \mu$$

Graph the force function versus θ with $\mu = 0.6$ and $W = 50$. Note that $\tan\theta = 0.6$ gives $\theta \approx 0.540$.